

SOME RECENT EXAMPLES OF COUNTABLY COMPACT GROUPS

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We will discuss the following constructions:

It is consistent that there exists a countably compact group whose size has countable cofinality (Proc. Amer. Math. Soc. 2003) This answered a question of van Douwen (Proc. Amer. Math. Soc. 1980).

If p and q are incomparable selective ultrafilters then there exist a p -compact group and a q -compact group whose product is not countably compact. (with S. Watson, Topology Appl. 2004). This answered Question 482 in the Open Problems in Topology.

For every $\kappa \leq 2^{\mathfrak{c}}$ there exists a group G such that G^α is countably compact for every $\alpha < \kappa$ and G^κ is not countably compact. (Fund. Math. 2005). This answered Question 477 in the Open Problems in Topology.

If there exists a selective ultrafilter p then there exists a p -compact group without non-trivial convergent sequences (with S. Garcia-Ferreira and S. Watson, Proc. Amer. Math. Soc. 2005) This shows that some form of Martin's Axiom is not necessary to construct a countably compact group without non-trivial convergent sequences. An MA example was obtained by van Douwen (Trans. Amer. Math. Soc. 1980) who argue why his construction seemed to need MA.

If there exists an infinite countably compact Abelian group without non-trivial convergent sequences then there exists a countably compact group whose square is not countably compact (Topology Appl. 2005). This give the "square version" of van Douwen's result in Trans Amer. Math. Soc. 1980. As a corollary, we obtain an example as Hart and van Mill's (Trans. Amer. Math. Soc.) under the existence of a selective ultrafilter.